

TRIGONOMETRIC FUNCTIONS

10. JAN . 2025

4 (a) Diberi $\sin x = a$, dengan sudut x ialah sudut cakah, cari dalam sebutan a , bagi

Given that $\sin x = a$, where angle x is an obtuse angle, find in terms of a , for

- (i) $\cos 2x$ * use formula 33
- (ii) $\tan(180^\circ - x)$ * use formula num 37

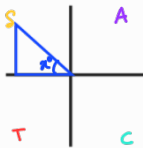
[3 markah]
[3 marks]

① If the keyword is "GIVEN" Draw triangle first!
* Before that, need to identify which quadrant

- ② Acute angle? ($< 90^\circ$)
1st quadrant
- Obtuse angle? ($90^\circ < \theta < 180^\circ$)
2nd quadrant
- Reflex angle? ($180^\circ < \theta < 360^\circ$)
3rd & 4th quadrant

③ Draw the triangle

* For this question, the triangle is at the 2nd quadrant

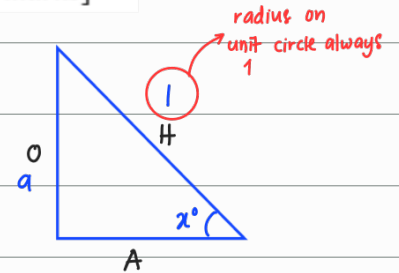


④ Must simplify / rewrite formulas in different way.

e.g. $\cos 2x$ can be written as:

- $\cos^2 x - \sin^2 x$ } will overcomplicate working
- $2\cos^2 x - 1$ }
- $1 - 2\sin^2 x$ } best choice since $\sin x$ is already given

$$\begin{aligned} \text{a) i) } \cos 2x &= 1 - 2\sin^2 x \quad \checkmark \text{ KI} \\ &= 1 - 2a^2 \quad \checkmark \text{ NI} \end{aligned}$$



$$\begin{aligned} \text{a) ii) } \tan(180^\circ - x) &= \frac{\tan 180^\circ - \tan x}{1 + \tan 180^\circ \tan x} \\ &= \frac{0 - \left(-\frac{a}{\sqrt{1-a^2}}\right)}{1 + 0\left(-\frac{a}{\sqrt{1-a^2}}\right)} \\ &= \frac{a}{\sqrt{1-a^2}} \quad \checkmark \text{ NI} \end{aligned}$$

$-\sqrt{1-a^2}$
remember to write -ve sign because the triangle is on the left side of y-axis

$$\circ \sec \theta = \frac{1}{\cos \theta}$$

$$\circ \tan \theta = \frac{\sin \theta}{\cos \theta}$$

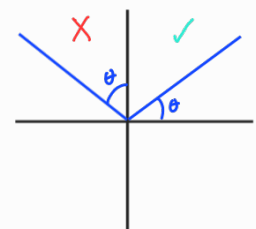
$$\circ \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

Tip: Focus on 3rd letter

$$\circ \cot \theta = \frac{1}{\tan \theta} \quad / \quad \frac{\cos \theta}{\sin \theta}$$

ALWAYS REMEMBER!

- Angle in a unit circle will always be attached to x-axis
- Write the -ve sign of a value as if writing coordinates
e.g. For left side, x value is -ve
For bottom part, y value is -ve
- If question didn't mention type of angle (acute/obtuse/reflex), there are 2 possible triangles in the unit circle. e.g. cosine is +ve in 1st & 4th quadrant



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① Always refer formula list while doing proving questions

In this case, refer formula

③②: $\sin 2A = 2 \sin A \cos A$

③③: $\sec^2 A = 1 + \tan^2 A$

② List out all other formulas that can help simplify the equation

o $\tan x = \frac{\sin x}{\cos x}$

o $\sec x = \frac{1}{\cos x}$

\Downarrow
 $\sec^2 x = \frac{1}{\cos^2 x}$

③ Try to simplify any "odd" trigo identity

e.g. since $\sin 2x = 2 \sin x \cos x$

" $\sec^2 x$ " does not belong in the equation

\therefore Have to change $\sec^2 x$ into something else

(b) (i) Buktikan bahawa:
Prove that: $\frac{2 \tan x}{1 + \tan^2 x} = \frac{\sin x}{\cos x}$ (WASIB HAFAL)
 $\rightarrow \sec^2 x$ (formula num 30)

[2 markah]
[2 marks]

LHS

$$\frac{2 \tan x}{1 + \tan^2 x} = \frac{2 \left(\frac{\sin x}{\cos x} \right)}{\sec^2 x} \rightarrow \frac{1}{\cos^2 x}$$

P1 (understanding in using different trigo identity formula)

$$= \frac{2 \sin x}{\cos x} \times \cos^2 x$$

$$= 2 \sin x \cos x \rightarrow \text{formula num 32}$$

$$= \sin 2x \text{ (shown) NI}$$

ALWAYS REMEMBER!

o Don't panic if you don't remember how to simplify fractions

\hookrightarrow use calculator to remember!

e.g. $2 \div \frac{1}{2} = 4$

this means; $2 \times \frac{2}{1} = 4$

} used in solving steps (line 2)

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(ii) Seterusnya, selesaikan persamaan $\frac{2 \tan x}{1 + \tan^2 x} = \frac{1}{2}$ untuk $0^\circ \leq \theta \leq 2\pi$ dan berikan jawapan dalam bentuk pecahan termudah dalam sebutan π rad.

Hence, solve the equation $\frac{2 \tan x}{1 + \tan^2 x} = \frac{1}{2}$ for $0^\circ \leq \theta \leq 2\pi$ and give your answer in the simplest fraction form in term of π rad.

[3 markah]
[3 marks]

Always sub in from previous question!

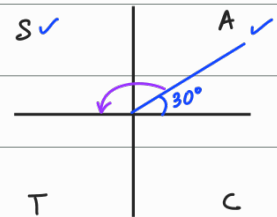
means previous answer needs to be applied back in, no extra working needed (understanding of previous question already seen by examiner)

* Do not do this question if you didn't do the previous one, marks will not be taken

$$\frac{2 \tan x}{1 + \tan^2 x} = \frac{1}{2}$$

$$\sin 2x = \frac{1}{2}$$

2 cycles, do not divide by 2 first to get x → last step



$\sin 2x = +ve$ value

∴ possible answers in 1st & 2nd quadrant

reference angle ← $\alpha = \sin^{-1}\left(\frac{1}{2}\right)$

$\alpha = 30^\circ$

1st cycle

2nd cycle

$2x = 30^\circ, (180^\circ - 30^\circ)$

$2x = (360^\circ + 30^\circ), (360^\circ + 150^\circ)$

$2x = 30^\circ, 150^\circ$

$2x = 390^\circ, 510^\circ$

NOW DIVIDE ALL ANGLES BY 2 TO FIND x

$2x = 30^\circ, 150^\circ, 390^\circ, 510^\circ$

$x = 15^\circ, 75^\circ, 195^\circ, 255^\circ$

Always arrange in ascending order

$x = \left(15^\circ \times \frac{\pi}{180^\circ}\right), \left(75^\circ \times \frac{\pi}{180^\circ}\right), \left(195^\circ \times \frac{\pi}{180^\circ}\right), \left(255^\circ \times \frac{\pi}{180^\circ}\right)$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

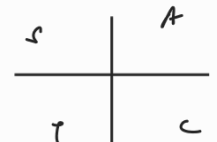
→ this step not necessary to show, no marks

TRIGO SOLVING QUESTIONS CAN BE GIVEN IN WHAT FORM?

general form (quadratic equation)
 $ax^2 + bx + c = 0$ use calc

similar coefficient / pekali ⇒ factorize
 $ax + xb = 0$
 $x(a+b) = 0$

* need to understand how to use the diagram



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(a) Buktikan bahawa $\frac{\cos^2 x}{1 - \sin x} = 1 + \sin x$.

Prove that $\frac{\cos^2 x}{1 - \sin x} = 1 + \sin x$.

$$\begin{aligned} \frac{\cos^2 x}{1 - \sin x} &= \frac{1 - \sin^2 x}{1 - \sin x} \quad \checkmark \text{PT} \\ &= \frac{(1 + \sin x)(1 - \sin x)}{1 - \sin x} \\ &= 1 + \sin x \quad \text{(shown)} \quad \checkmark \text{K1} \end{aligned}$$

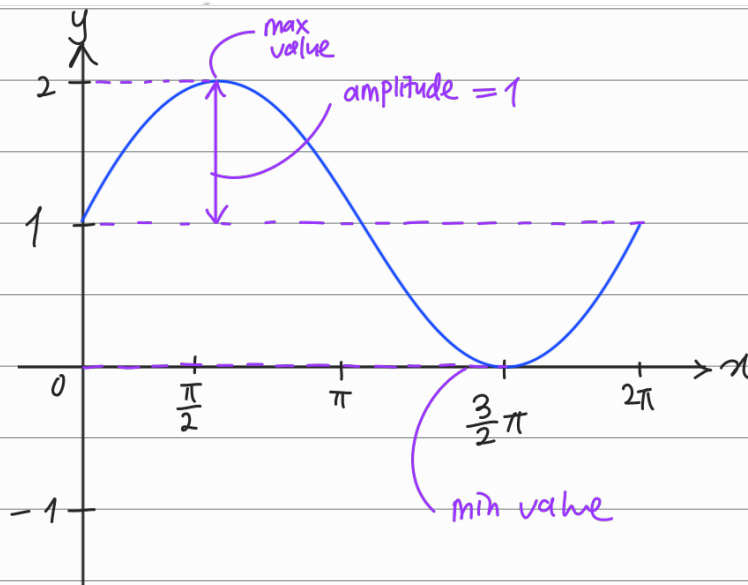
amplitude (height)
 $y = a \sin bx + c$
 ↓
 How many cycles in 360°?
 ↓
 makes graph go up/down

o maximum value = $a + c$

o minimum value = $-a + c$

(b) Lakarkan graf $y = 1 + \sin x$ untuk $0 \leq x \leq 2\pi$.

Sketch the graph of $y = 1 + \sin x$ for $0 \leq x \leq 2\pi$.



① Determine shape of graph



② Find how many cycles for 360°

③ Determine values on x -axis

* Sometimes questions ask for x -axis to be less than 360°

ALWAYS REMEMBER!

Always go through question carefully, the range of x ($0 \leq x \leq 2\pi$) is also equivalent to ($0 \leq x \leq 360^\circ$) BUT YOU WILL NOT GET MARKS FOR THE VALUES ON YOUR x -AXIS

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(c) Seterusnya, dengan menggunakan paksi yang sama, lukiskan garis lurus yang sesuai untuk mencari bilangan penyelesaian bagi persamaan $\frac{\cos^2 x}{1 - \sin x} + 1 = \frac{x}{\pi}$ untuk $0 \leq x \leq 2\pi$. Nyatakan bilangan penyelesaian itu.

Hence, by using the same axes, draw a suitable straight line to find the number of solutions of the equation $\frac{\cos^2 x}{1 - \sin x} + 1 = \frac{x}{\pi}$ for $0 \leq x \leq 2\pi$. State the number of solutions.

[3 markah]

[3 marks]

Always
sub in from
previous question

can substitute
with y

SOMETIMES the line used to find the number of solutions is not a "straight line"

HOW TO IDENTIFY:

- question will not mention "draw a suitable straight line"
- $y = mx + c$ (straight line)
e.g. $y = \frac{x}{\pi}$, $y = \frac{x}{2\pi}$, $y = \pi x$
- $y = ?$ (straight line)
e.g. $y = 2$, $y = 4$
- $y = \frac{1}{x}$ (reciprocal / curve)
e.g. $y = \frac{\pi}{x}$, $y = \frac{\pi}{2x}$, $y = \frac{2\pi}{x}$

$$y + 1 = \frac{x}{\pi}$$

Plot:

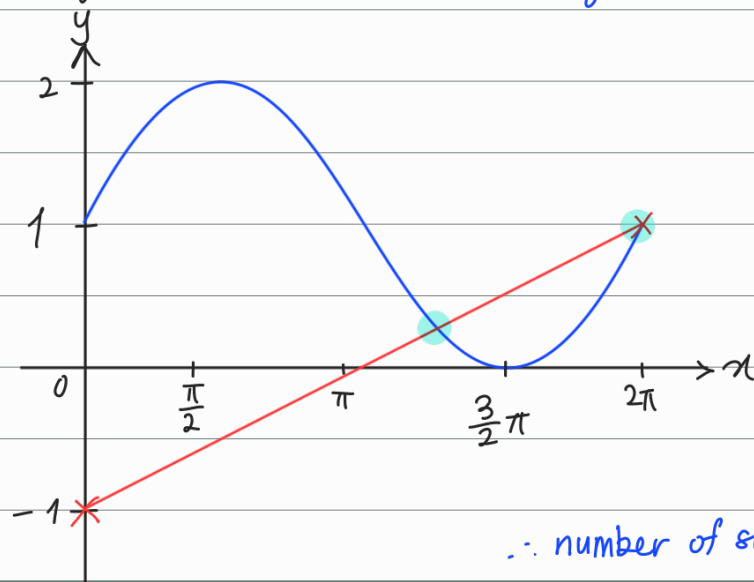
x	0	2π
y	-1	1

$$y = \frac{x}{\pi} - 1$$

$$y = \frac{0}{\pi} - 1 \quad y = \frac{2\pi}{\pi} - 1$$

$$y = -1 \quad y = 1$$

equation for
straight line



\therefore number of solutions = 2

ALWAYS REMEMBER!

- Leave extra space while doing question (b) because you might need extra space to plot the new line
- Make sure your curve / straight line is smooth, don't erase multiple times (It might be visible in the pdf of the exam paper)